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Use the Method of induction to prove that $\sum_{r=1}^{n} 2^{r-1} = 2^n - 1$

Soln

Put $n = 1$

$L.H.S = \sum_{r=1}^{1} 2^{r-1} = 2^{1-1} = 2^0 = 1$

$R.H.S = 2^1 - 1 = 2 - 1 = 1$

$\therefore$ it is true for $n = 1$

Suppose that it is also true for $n = K$

$$\sum_{r=1}^{K} 2^{r-1} = 1 + 2 + 4 + \cdots + 2^{K-1} = 2^K - 1 \quad \cdots \text{①}$$

For $n = K+1$

Here, you add the next term as follows:

$2^{r-1} = 2^{K+1-1} = 2^K$ is the next term to add to both sides of equ/statement ① above.

$$\sum_{r=1}^{K+1} 2^{r-1} = (1 + 2 + 4 + \cdots + 2^{K-1}) + 2^K$$

$$= \sum_{r=1}^{K} 2^{r-1} + 2^K \quad \cdots \text{②}$$

But $\sum_{r=1}^{K} 2^{r-1} = 2^K - 1$ as stated in ①

$$\therefore \sum_{r=1}^{K+1} = (2^K - 1) + 2^K = 2^K + 2^K - 1$$

$$= 2^1(2^K) - 1$$

$$= 2^{K+1} - 1 = 2^n - 1 \quad (n = K+1)$$

Hence the statement is true for $n = 1, 2, 3, \cdots, K$, and $K+1$

$\therefore \sum_{r=1}^{n} 2^{r-1} = 2^n - 1$ as required

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